Lower bound on the magnetic field strength in the hot universe

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It is assumed that long range coherent magnetic fields in the universe were spontaneously generated at high temperature due to vacuum polarization of non-Abelian gauge fields, and resulted in the present intergalaxy magnetic field. The zero value of the screening mass for fields of this type was discovered recently. Here, a procedure to estimate the field strengths at different temperatures is developed and the lower bound on the magnetic field strength $B \sim 10^{14} G$, at the electroweak phase transition temperature, is derived. As a particular case, the standard model is considered. Some model dependent peculiarities of the phenomena under investigation are briefly discussed.

I. INTRODUCTION

Intergalactic magnetic fields are among the most interesting discoveries in modern cosmology. Recently, lower bounds of the order of $B \sim 10^{-15} G$ have been established observationally [1, 2] and the search for the origin of these fields has intensified. One of the best candidates are clearly primordial fluctuations. But there is also a number of other candidates (for a review, see [3]).

In this paper we would like to discuss some different mechanism, based on non-Abelian magnetic fields. As it was shown recently, a spontaneous magnetization appears in non-Abelian gauge theories at high temperature. This was found by analytic methods in [4]–[7] and it was confirmed by lattice simulations in [8]. The basic idea rests on the known observation that in non-Abelian gauge theories at high temperature a spontaneous vacuum magnetization occurs. This is the consequence of the spectrum of a color charged gluon,

$$p_0^2 = p_{||}^2 + (2n+1)gB \qquad (n = -1, 0, 1, ...), \quad (1)$$

in a homogeneous magnetic background, B, $p_{||}$ is a momentum directed along the field. Here, a tachyonic mode is present in the ground state ($n = -1$). In fact, one observes that $p_0^2 < 0$ resulting from the interaction of the magnetic moment of the spin-1 field with the magnetic field. This phenomenon was first observed by Savvidy [9] at zero temperature, $T = 0$, and got known as the Savvidy vacuum. However, at zero and low temperature this state is not stable. It decays under emission of gluons until the magnetic field B disappears. The picture changes with increasing temperature where a stabilization sets in. This stabilization is due to vacuum polarization and it depends on two dynamical parameters. These are a magnetic mass of the color charged gluon and

an A_0-condensate, which is proportional to the Polyakov loop [10]. This configuration is perfectly stable, since its energy is below the perturbative one and the minimum is reached for a field of order $gB \sim g^4 T^2$. Although the phenomenon was discovered in $SU(2)$ gluodynamics, it is common to other $SU(N)$ gauge fields which can be used to extend the standard $(SU(2) \times U(1))_{ew} \times SU(3)_c$ model of elementary particles.

An important property of such temperature dependent magnetic fields is the vanishing of their magnetic mass, $m_{magn} = 0$. This was found both in one-loop analytic calculations [11] and also in lattice simulations [12]. The mass parameter describes the inverse spatial scales of the transverse field components, similarly to the Debye mass m_D, related to the inverse space scale for the electric (Coulomb) component. The absence of a screening mass means that the spontaneously generated Abelian chromomagnetic fields are long range at high temperature, as is common for the $U(1)$ magnetic field. Hence, it is reasonable to believe that at each stage of the evolution of the hot universe, spontaneously created, strong, long-range magnetic fields of different types have been present. Owing to the property of being unscreened, they have influenced various processes and phase transitions.

The dependence on the temperature of these fields differs from that of the typical $U(1)$ magnetic fields. Recall that, in the latter case, the magnetic (in fact, hyper-magnetic) field, created by some specific mechanism, is implemented in a hot plasma and decreases according to the law $B \sim T^2$, which is a consequence of magnetic flux conservation (see, for instance, [3]). However, in the case of spontaneous vacuum magnetization the magnetic flux is *not* conserved. Instead, a specific flux value is generated at each temperature. This fact has to be taken into consideration when the cooling pattern of the hot non-Abelian plasma is investigated. This also concerns the $SU(2)_{ew}$ component of the electromagnetic field.

In the present paper we estimate the strength of the magnetic field at the temperatures of the electroweak T_c^{ew} phase transition, assuming that this field was spontaneously generated by a mechanism as described above.

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Although such phenomenon is nonperturbative, we carry out an actual calculation in the framework of a consistent effective potential (EP) accounting for the one-loop, $V^{(1)}$, and the daisy (or ring), V^{ring}, diagram contributions of the standard model. In Sect. 2 we qualitatively describe, in more detail, some important aspects of the investigated phenomena. In Sect. 3 the EP of an Abelian constant electromagnetic B field at finite temperature is obtained. It is then used, in Sect. 4, to estimate the magnetic field strength at the electroweak phase transition temperatures. A discussion of the results obtained in the paper together with some prospects for further work are provided in Sect. 5.

II. QUALITATIVE CONSIDERATION

In this section we describe, in a qualitative manner, the most relevant aspects of the phenomena considered. All of them follow from very basic asymptotic freedom and spontaneous symmetry breaking considerations at finite temperature. Our main assumption is that the intergalactic magnetic field has been spontaneously created at high temperature. We believe this to be a quite reasonable idea because, physically, the magnetization is the consequence of a large magnetic moment for charged non-Abelian gauge fields (let us just remind of the gyromagnetic ratio $\gamma = 2$ for W-bosons). This property eventually results in the asymptotic freedom of the model in the presence of external fields. We will discuss the procedure to relate the present value of the intergalactic magnetic field with the one generated in the restored phase.

First, we note that in non-Abelian gauge theories magnetic flux conservation does not hold. This is due to spontaneous vacuum magnetization which depends on the temperature. The vacuum acts as a specific source generating classical fields. Second, the magnetization is strongly dependent on the scalar field condensate present in the vacuum at low temperature. This point was investigated at zero temperature by Goroku [13]. For finite temperature, it is considered in the present paper for the first time. The observation is that, in both cases, the spontaneous vacuum magnetization takes place for a small scalar field $\phi \neq 0$, only. For the values of ϕ corresponding to any first order phase transition it does not happen. This means that, after the electroweak phase transition occurs, the vacuum polarization ceases to generate magnetic fields and magnetic flux conservation holds again. As a result, the familiar dependence on the temperature $B \sim T^2$ is restored.

Another aspect of the problem is the composite structure of the electromagnetic field A_μ. The potentials read

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g' A_\mu^3 + g b_\mu),$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g A_\mu^3 - g' b_\mu), \qquad (2)$$

where Z_μ is the Z-boson potential, A_μ^3, b_μ are the Yang-Mills gauge field third projection in the weak isospin space and the potential of the hypercharge gauge fields, respectively, and g and g' are correspondingly $SU(2)$ and $U(1)_Y$ couplings. After the electroweak phase transition, the Z-boson acquires mass and the field is screened. Since the hypermagnetic field is not spontaneously generated, at high temperature only the component $A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} g' A_\mu^3 = \sin\theta_w A_\mu^3$ is present. Here θ_w is the Weinberg angle, $\tan\theta_w = \frac{g'}{g}$. This is the only component responsible for the intergalactic magnetic field at low temperature.

In the restored phase the field $b_\mu = 0$, and the complete weak-isospin chromomagnetic field $A_\mu^{(3)}$ is unscreened. This is because the magnetic mass of this field is zero [12]. Thus, the field is a long range one. It provides the coherence length to be sufficiently large. After the phase transition, part of the field is screened by the scalar condensate. In the restored phase, the constituent of the weak isospin field corresponding to the magnetic one is given by the expression

$$B(T) = \sin\theta_w(T) B^{(3)}(T), \qquad (3)$$

where $B^{(3)}(T)$ is the strength of the field generated spontaneously.

To relate the present value of the intergalactic magnetic field with the field which existed before the electroweak phase transition, we take into consideration that, after the phase transition, the spontaneous vacuum magnetization does not occur. Therefore, for the electroweak transition temperature T_{ew} we can write:

$$\frac{B(T_{ew})}{B_0} = \frac{T_{ew}^2}{T_0^2} = \frac{\sin\theta_w(T_{ew}) B^{(3)}(T_{ew})}{B_0}. \qquad (4)$$

Here B_0 is the present value of the intergalaxy magnetic field strength $B_0 \sim 10^{-15} G$. The left-hand-side relates the value $B(T_{ew})$ with B_0. The right-hand-side allows to express the weak isospin magnetic field in the restored phase through B_0, knowing the temperature dependence of the Weinberg angle $\theta_w(T)$. This relation contains an arbitrary temperature normalization parameter τ. It can be fixed for given temperature and B_0. After that, the field strength values at various temperatures can be calculated. In particular, the total weak isospin field strength is given by the sum $\cos\theta_w(T_{ew}) B^{(3)}(T_{ew}) + B(T_{ew})$.

An important aspect of this scenario is that the precise nature of the till now unknown theory extending the standard model is not very important for estimating the field strength B at temperatures close to T_{ew}. This is because any new gauge field of the extended model in question will be screened at the relevant higher temperatures corresponding to the spontaneous symmetry breaking of some basic symmetries. At high temperatures, when these symmetries are restored, related magnetic fields emerge. Using these ideas the value of the field

strength at the Planck era has been estimated by Pollock [14]. Summing up, we conclude that our estimate here gives a lower bound on the magnetic field strength for the hot universe.

III. EFFECTIVE POTENTIAL AT HIGH TEMPERATURE

As we noted above, spontaneous vacuum magnetization and the absence of a magnetic mass for the Abelian magnetic fields are nonperturbative effects to be precisely determined, in particular, in lattice simulations [8, 12]. The main conclusions of these investigations are that a stable magnetized vacuum does exist at high temperature and that the magnetic mass of the created field is zero. Concerning the actual value of the field strength, it is close to the one calculated within the consistent effective potential which takes into account one-loop plus daisy diagrams. Thus, in the present investigation we restrict ourselves to that approximation. The main purpose for doing this is to be able to develop analytic calculations, in order to clarify the results obtained.

The complete EP for the standard model is given in the review [15]. In the present investigation we are interested in two different limits:

1. The weak magnetic field and large scalar field condensate, $h = eB/M_w^2 < \phi^2, \phi = \phi_c/\phi_0, \beta = 1/T$.

2. The case of restored symmetry, $\phi = 0, gB \neq 0, T \neq 0$.

For the first case we show the absence of spontaneous vacuum magnetization at finite temperature. For the second one we estimate the field strength at high temperature. Here M_w is the W-boson mass at zero temperature, ϕ_c is a scalar field condensate, and ϕ_0 its value at zero temperature.

To demonstrate the first property we consider the one-loop contribution of W-bosons (see also Eq. (27) of Ref. [6]),

$$V_w^{(1)}(T,h,\phi) = \frac{h}{\pi^2\beta^2}\sum_{n=1}^{\infty}\Big[\frac{(\phi^2-h)^{1/2}\beta}{n}K_1(n\beta(\phi^2-h)^{1/2})$$
$$-\frac{(\phi^2+h)^{1/2}\beta}{n}K_1(n\beta(\phi^2+h)^{1/2})\Big]. \quad (5)$$

Here n labels discrete energy values and $K_1(z)$ is the MacDonald function.

The main goal of our investigation is the restored phase of the standard model. To this end, we deduce the high temperature contribution of the complete effective potential relevant for this case using the results in Ref. [15]. First, we write down the one-loop W-boson contribution as the sum of the pure Yang-Mills weak-isospin part ($\tilde{B} \equiv B^{(3)}$),

$$V_w^{(1)}(\tilde{B},T) = \frac{\tilde{B}^2}{2} + \frac{11}{48}\frac{g^2}{\pi^2}\tilde{B}^2\log\frac{T^2}{\tau^2} - \frac{1}{3}\frac{(g\tilde{B})^{3/2}T}{\pi}$$

$$-i\frac{(g\tilde{B})^{3/2}T}{2\pi} + O(g^2\tilde{B}^2), \quad (6)$$

where τ is a temperature normalization point, and the charged scalars [6],

$$V_{sc}^{(1)}(\tilde{B},T) = -\frac{1}{96}\frac{g^2}{\pi^2}\tilde{B}^2\log\frac{T^2}{\tau^2} + \frac{1}{12}\frac{(g\tilde{B})^{3/2}T}{\pi} + O(g^2\tilde{B}^2), \quad (7)$$

describing the contribution of longitudinal vector components. The first term in Eq. (6) is the tree-level energy of the field. This representation is convenient for the case of extended models including other gauge and scalar fields. Depending on the specific case, one can take into consideration the parts (6) or (7), correspondingly. In the standard model, the contribution of Eq. (7) has to be included with a factor 2, due to two charged scalar fields entering the scalar doublet of the model. In the case of the Two-Higgs-Doublet standard model, this factor must be 4, etc. The imaginary part is generated because of the unstable mode in the spectrum (1). It is canceled by the term appearing in the contribution of the daisy diagrams for the unstable mode [7],

$$V_{unstable} = \frac{g\tilde{B}T}{2\pi}[\Pi(\tilde{B},T,n=-1) - g\tilde{B}]^{1/2} + i\frac{(g\tilde{B})^{3/2}T}{2\pi}. \quad (8)$$

Here $\Pi(\tilde{B},T,n=-1)$ is the mean value for the charged gluon polarization tensor taken in the ground state $n = -1$ of the spectrum (1). If this value is sufficiently large, spectrum stabilization due to the radiation correction takes place. This possibility formally follows from the temperature and field dependences of the polarization tensor in the high temperature limit $T \to \infty$ [16]: $\Pi(\tilde{B},T,n=-1) = c\,g^2T\sqrt{g\tilde{B}}$, where $c > 0$ is a constant which must be calculated explicitly. At high temperature the first term can be larger then $g\tilde{B}$.

From Eqs. (6) and (8) it follows that the imaginary part cancels. Hence, we see that taking rings into account leads to vacuum stabilization even if $\Pi(\tilde{B},T,n=-1)$ is smaller then $g\tilde{B}$. Actually, in the latter case, the imaginary part will be smaller than in Eq. (6).

The high temperature limit of the fermion contribution looks as follows,

$$V_{fermion} = -\frac{\alpha}{\pi}\sum_f \frac{1}{6}q_f^2\tilde{B}^2\log\frac{T}{\tau}, \quad (9)$$

where the sum is extended to all leptons and quarks, and q_f is the fermion electric charge in positron units. Hence, it follows that in the restored phase all the fermions give the same contribution.

Now, let us present the EP for ring diagrams describing the long range correlation corrections at finite temperature [17, 18],

$$V_{ring} = \frac{1}{24\beta^2}\Pi_{00}(0) - \frac{1}{12\pi\beta}Tr[\Pi_{00}(0)]^{3/2}$$
$$+ \frac{(\Pi_{00}(0))^2}{32\pi^2}\Big[\log\big(\frac{4\pi}{\beta(\Pi_{00}(0))^{1/2}}\big) + \frac{3}{4} - \gamma\Big], \quad (10)$$

where the trace means summation over all the contributing states, $\Pi_{00} = \Pi_\phi(k = 0, T, B)$ for the Higgs particle; $m_D^2 = \Pi_{00} = \Pi_{00}(k = 0, T, B)$ are the zero-zero components of the polarization functions of gauge fields in the magnetic field taken at zero momenta, called the Debye mass squared, γ is Euler's gamma. These terms are of order $\sim g^3(\lambda^{3/2})$ in the coupling constants. The detailed calculation of these functions is given in Ref. [15]. We give the results for completeness,

$$\Pi_\phi(0) = \frac{1}{24\beta^2}\left[6\lambda + \frac{6e^2}{\sin^2(2\theta_w)} + \frac{3e^3}{\sin^2\theta_w}\right]$$
$$+ \frac{2\alpha}{\pi}\sum_f\left(\frac{\pi^2 K_f}{3\beta^2} - |q_f B|K_f\right)$$
$$+ \frac{(eB)^{1/2}}{8\pi\sin^2\theta_w\beta}e^2 3\sqrt{2}\zeta\left(-\frac{1}{2}, \frac{1}{2}\right). \qquad (11)$$

Here $K_f = \frac{m_f^2}{m_w^2} = \frac{G_{Yukawa}^2}{g^2}$ and λ is the scalar field coupling. The terms $\sim T^2$ give standard contributions to the temperature mass squared coming from the boson and fermion sectors. The B-dependent terms are negative (the value of $3\sqrt{2}\zeta(-\frac{1}{2}, \frac{1}{2}) = -0.39$). They decrease the value of the screening mass at high temperature. The Debye masses squared for the photons, Z-bosons and neutral current contributions are, correspondingly,

$$m_{D,\gamma}^2 = g^2\sin^2\theta_w\left[\frac{1}{3\beta^2} + O(eB\beta^2)\right],$$
$$m_{D,Z}^2 = g^2\left(\tan^2\theta_w + \frac{1}{4\cos^2\theta}\right)\left[\frac{1}{3\beta^2} + O(eB\beta^2)\right],$$
$$m_{D,neutral}^2 = \frac{g^2}{8\cos^2\theta_w\beta^2}(1 + 4\sin^4\theta_w) + O(eB\beta^2). \quad (12)$$

As one can see, the dependence on B appears at order $O(T^{-2})$.

The W-boson contributions to the Debye mass of the photons is

$$m_{D,W}^2 = 3g^2\sin^2\theta_w\left[\frac{1}{3\beta^2} - \frac{(g\sin\theta_w B)^{1/2}}{2\pi\beta}\right]. \quad (13)$$

An interesting feature of this expression is the negative sign of the next-to-leading terms dependent on the field strength. Finally, we give the contribution of the high temperature part in Eq. (8) $\Pi(\tilde{B}, T, n = -1)$ [15],

$$\Pi(\tilde{B}, T, n = -1) = \alpha\left[12.33\frac{(g\sin\theta_w B)^{1/2}}{\beta}\right.$$
$$\left. + 4i\frac{(g\sin\theta_w B)^{1/2}}{\beta}\right]. \quad (14)$$

This expression has been calculated from the one-loop W-boson polarization tensor in the external field at high temperature. It contains the imaginary part which comes from the unstable mode in the spectrum (1). Its value is small, as compared to the real one. It is of the order

of the usual damping constants in plasma at high temperature. Thus, it will be ignored in actual calculations, in what follows. In fact, this part should be calculated in a more consistent scheme which starts with a regularized stable spectrum. On the other hand, as we noted above, the stability problem is a non-perturbative one. The stabilization can be realized not only by the radiation corrections but also by some other mechanisms. For example, due to A_0 condensation [4] at high temperature. We observed the stable vacuum state in the lattice simulations [8]. Therefore, we do believe that this problem has a positive solution. Summing up, we now have all what is necessary in order to investigate in depth the problem of interest.

IV. MAGNETIC FIELD STRENGTH AT T_{ew}

We will now show that spontaneous vacuum magnetization does not occur at finite temperature and for non-small values of the scalar field condensate $\phi \neq 0$. To this end we notice that the magnetization is produced by the gauge field contribution, given by Eq. (5). So, we consider the limit of $\frac{gB}{T^2} \ll 1$ and $\phi^2 > h$. For this case we use the asymptotic expansion of $K_1(z)$,

$$K_1(z) \sim \sqrt{\frac{\pi}{2z}}e^{-z}\left(1 + \frac{3}{8z} - \frac{15}{128z^2} + \ldots\right), \qquad (15)$$

where $z = n\beta(\phi^2 \pm h)^{1/2}$. We now investigate the limit $\beta \to \infty, \frac{T}{\phi} \ll 1$. The leading contribution is then given by the first term of the temperature sum in Eq. (5). We can also substitute $(\phi^2 \pm h)^{1/2} = \phi(1 \pm \frac{h}{2\phi^2})$. In this approximation, the sum of the tree level energy and the contribution (5) reads

$$V = \frac{h^2}{2} - \frac{h^2}{\pi^{3/2}}\frac{T^{1/2}}{\phi^{1/2}}\left(1 - \frac{T}{2\phi}\right)e^{-\phi/T}. \qquad (16)$$

The second term is exponentially small and the stationary equation $\frac{\partial V}{\partial h} = 0$ admits the trivial solution $h = 0$. This estimate can be easily verified in numeric calculation of the total effective potential. Hence, we conclude that, as at zero temperature [13], after symmetry breaking the vacuum spontaneous magnetization does not take place.

To estimate the magnetic field strength in the restored phase at the electroweak phase transition temperature, the total effective potential obtained in the previous section must be used and the parameters entering Eq. (4) need to be calculated. This can be best done numerically. Specifically, we consider here the contribution to this potential accounting for the one-loop W-boson terms. The high temperature expansion for the EP coming from charged vector fields is given in Eq. (6). Assuming stability of the vacuum state, we calculate the value of the chromomagnetic weak isospin field spontaneously

generated at high temperature from Eqs. (6) and (7):

$$\tilde{B}(T) = \frac{1}{16}\frac{g^3}{\pi^2}\frac{T^2}{\left(1 + \frac{5}{12}\frac{g^2}{\pi^2}\log\frac{T}{\tau}\right)^2}. \qquad (17)$$

We relate this expression with the intergalactic magnetic field B_0.

Let us introduce the standard parameters and definitions, $\frac{g^2}{4\pi} = \alpha_s, \alpha = \alpha_s\sin^2\theta_w, \frac{(g')^2}{4\pi} = \alpha_Y$ and $\tan^2\theta_w(T) = \frac{\alpha_Y(T)}{\alpha_s(T)}$, where α is the fine structure constant. To find the temperature dependence of the Weinberg angle, the behavior of the hypercharge coupling g' on the temperature has to be computed. From Eq. (7) it follows that this behavior is nontrivial. The logarithmic temperature-dependent term is negative. But, as is well known, in asymptotically free models this sign will unavoidably be changed to a positive value due to the contributions of other fields. This particular value is model dependent and we will not calculate it in this paper. Instead, for a rough estimate, we replace it with the zero temperature number: $\sin^2\theta_w(T) = \sin^2\theta_w(0) = 0.23$.

For the given temperature of the electroweak phase transition, T_{ew}, the magnetic field is

$$B(T_{ew}) = B_0\frac{T_{ew}^2}{T_0^2} = \sin\theta_w(T_{ew})\tilde{B}(T_{ew}). \qquad (18)$$

Assuming $T_{ew} = 100GeV = 10^{11}eV$ and $T_0 = 2.7K = 2.3267\cdot 10^{-4}eV$, we obtain

$$B(T_{ew}) \sim 1.85\ 10^{14}G. \qquad (19)$$

This value can be considered as a lower bound on the magnetic field strength at the electroweak phase transition temperature.

Hence, for the value of $X = \log\frac{T_{ew}}{\tau}$, we have the equation

$$B_0 = \frac{1}{2}\frac{\alpha^{3/2}}{\pi^{1/2}\sin^2\theta_w}\frac{T_0^2}{\left(1 + \frac{5\alpha}{3\pi\sin^2\theta_w}X\right)^2}. \qquad (20)$$

Since all the values here are known, $\log\tau$ can be estimated. After that the field strengths at different higher temperatures can be found. Of course, our estimate is a rough one because of having ignored the temperature dependence of the Weinberg angle. To guess the value of the parameter τ we take the field strength $B_0 \sim 10^{-9}G$, usually used in cosmology (see, for example, [14]). In this case, from Eq. (20) we obtain $\tau \sim 300eV$. For the lower bound value $B \sim 10^{-15}G$ this parameter is much smaller. The strong suppression of the field strength is difficult to explain within the standard model. This point will be discussed below.

To take into consideration the fermion contribution Eq. (9), we have to substitute the expression $\frac{5}{12}\frac{g^2}{\pi^2}\log\frac{T}{\tau}$

in Eqs. (17) and (20) with the value

$$\left(\frac{5}{3} - \sum_f\frac{1}{6}q_f^2\right)\frac{\alpha_s}{\pi}\log\frac{T}{\tau}. \qquad (21)$$

In the above estimate, we have taken into account the one-loop part of the EP of order $\sim g^2$ in the coupling constant. The ring diagrams are of order $\sim g^3$ and provide a small numeric correction to this result in the high temperature approximation. As it was mentioned before, had we taken into account all the terms listed in the previous section the results would have not changed essentially.

The field strength at higher temperatures depends on the particular model extending the standard one. Spontaneous vacuum magnetization in the minimal supersymmetric standard model has been investigated in Ref. [18]. The field strength generated in this model is smaller, as compared with the situation here considered. Pollock [14] has investigated this problem for the case of the Planck era, where magnetic fields of order $B \sim 10^{52}G$ have been estimated. We will further discuss these results in the concluding section.

V. DISCUSSION

Here we summarize our main results. The key issue in the problem under investigation is the spontaneous magnetization of the vacuum, which eliminates the magnetic flux conservation principle at high temperatures. This vacuum polarization is responsible for the value of the field strength $B(T)$ at each temperature and serves as a source for it. We have also shown that, at finite temperature and after the symmetry breaking, a scalar field condensate suppresses the magnetization. Hence it follows that the actual nature of the particular extended model is not essential at sufficiently low temperatures when the decoupling of heavy gauge fields has happened already. These statements are new and come as an interesting surprise, as compared with the standard notions based on the ubiquitous scenario with magnetic flux conservation. In the latter case one assumes that the magnetic field is created by some mechanism at different stages of the universe evolution. Then the temperature dependence $(B \sim T^2)$ is regulated by magnetic flux conservation, only.

The present value of the intergalactic magnetic field is related in our model with the field strengths at high temperatures in the restored phase. Because of the zero magnetic mass for Abelian magnetic fields, as discovered recently [12], there is no problem in the generation of fields having a large coherence length. Knowing the particular properties of the extended model it is possible to estimate the field strengths at any temperature. This can be done for different schemes of spontaneous symmetry breaking (restoration) by taking into account the fact that, after the decoupling of some massive gauge fields,

the corresponding magnetic fields are screened. Thus, the higher the temperature is the larger will be the number of strong long range magnetic fields of different types that will be generated in the early universe.

Now, let us compare our results with those of Ref. [14], where spontaneous vacuum magnetization at high temperature was applied to estimate the field strength at the Planck era. In that paper, in order to estimate the field strength, heterotic superstring theory $E_8 \times E_8'$ was considered as a basic ingredient. At the Planck era, the magnetic field strength has been estimated to be of order $\sim 10^{52}G$. In contrast to our considerations, it was assumed there that the magnetic field approximately scales as $B \sim T^2$. That is, vacuum magnetization was taken into account only at the very first moments of the universe evolution. Further, recent results implying a zero magnetic mass for the Abelian chromomagnetic fields also change the picture of the magnetized early universe substantially. According to those, the created magnetic fields existed already on the horizon scales. They were switched off at some mass scales, because of the spontaneous symmetry breaking as the temperature was lowering and the decoupling of heavy gauge fields occured. As a result, at the electroweak phase transition only the component $B^{(3)}$ of the $SU(2)$ weak isospin group remains unscreened and eventually results in the present day intergalactic magnetic field. The processes of decoupling were also not taken into consideration in Ref. [14]. Thus, it was impossible there to relate the electromagnetic field B_0 with the magnetic fields generated at high temperatures.

Our analysis has shown that, at the electroweak phase transition temperature, magnetic fields of the order $B(T_{ew}) \sim 10^{14}G$ were present. To estimate the field strengths at high temperatures, one needs to put into play a number of characteristic features of the standard model and its particular extension. First, we note that quarks possess both electric and color charges. Therefore, there is a mixing between the color and usual magnetic fields owing to the quark loops. Second, there are peculiarities related with the particular content of the extended model considered. For example, in the Two-Higgs-Doublet standard model the contribution $\sim (gB)^{3/2}T$ in Eq. (6) is exactly canceled by the corresponding term in Eq. (7), because of the four charged scalar fields entering the model. They interact with gauge fields with the same coupling constant. However, in this model the doublets interact differently with fermions. This changes the effective couplings of the doublets with gauge fields and results in non-complete cancelations. As a result, a strong suppression of the spontaneously created magnetic field is expected in this model. This, in principle, could explain a very small value of the intergalactic magnetic field at low temperature. There are other peculiarities which influence the high temperature phase of the universe. They require further investigations, which we leave for a future publication.

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